Statistical Measures
for
Evaluation of Methods
for
Syndromic Surveillance

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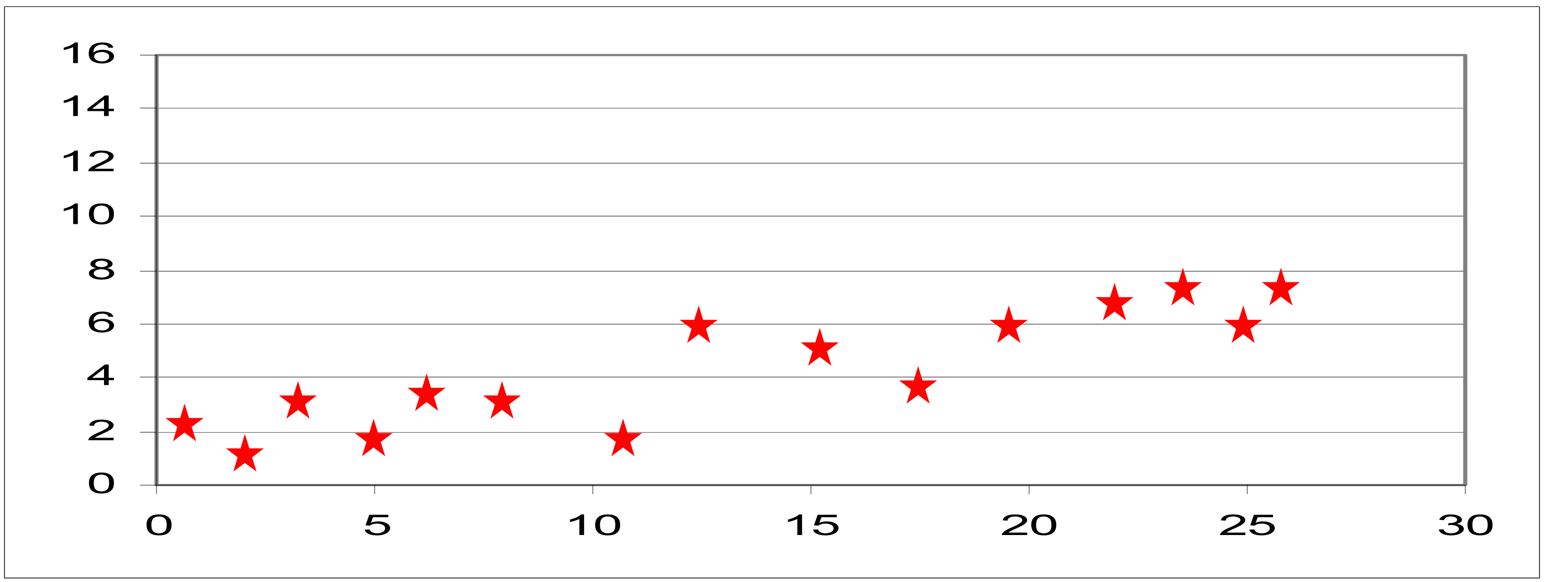
Statistical methods
to separate important changes from stochastic variation.



Enough information for decision?

Surveillance

- **Repeated measurements**
- **Repeated decisions**
- **No fix hypothesis**
- **Time important**

Time important

- **Methods**

- **Evaluations**

  – Frisén, M. (2003), Statistical Surveillance. Optimality and Methods., *International Statistical Review,*71, 403-434.

Evaluations

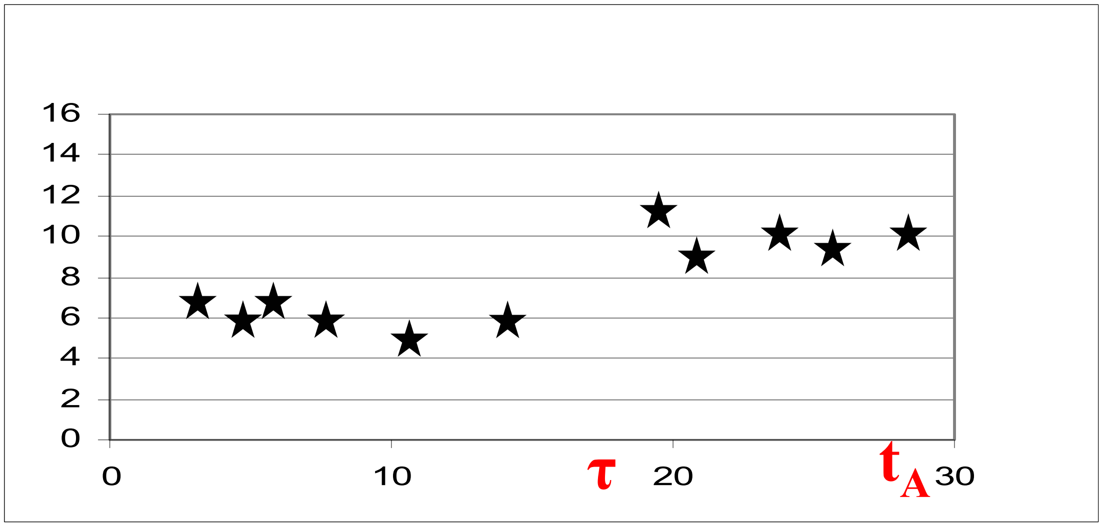
- Quick detection
- Few false alarms

- Frisén, M. (1992). Evaluations of methods for statistical surveillance. *Statistics in Medicine,* 11, 1489 - 1502**.**

Change in distribution

The First $(\tau-1)$ observations $x_{\tau-1} = x(1), ..., x(\tau-1)$ have density f^D

The following observations have density f^C



Alarm

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Syndromic Surveillance NY 2003

False alarms

- The Average Run Length at no change, $ARL^0 = E(t_A | D)$
- The false alarm probability $P(t_A < \tau)$

Delay

- $ED(t) = E[\max(0, \mathbf{t_A} - \mathbf{t}) \mid \tau = t]$
- The expected delay depends on the time τ of the change
- ED = Expected Delay = $E_\tau[ED(\tau)]$
- ARL[1] = ED(1) The Average Run Length until detection of a change (that occurred at the same time as the inspection started). $E(t_A \mid \tau = 1)$
- **P**robability of **S**uccessful **D**etection

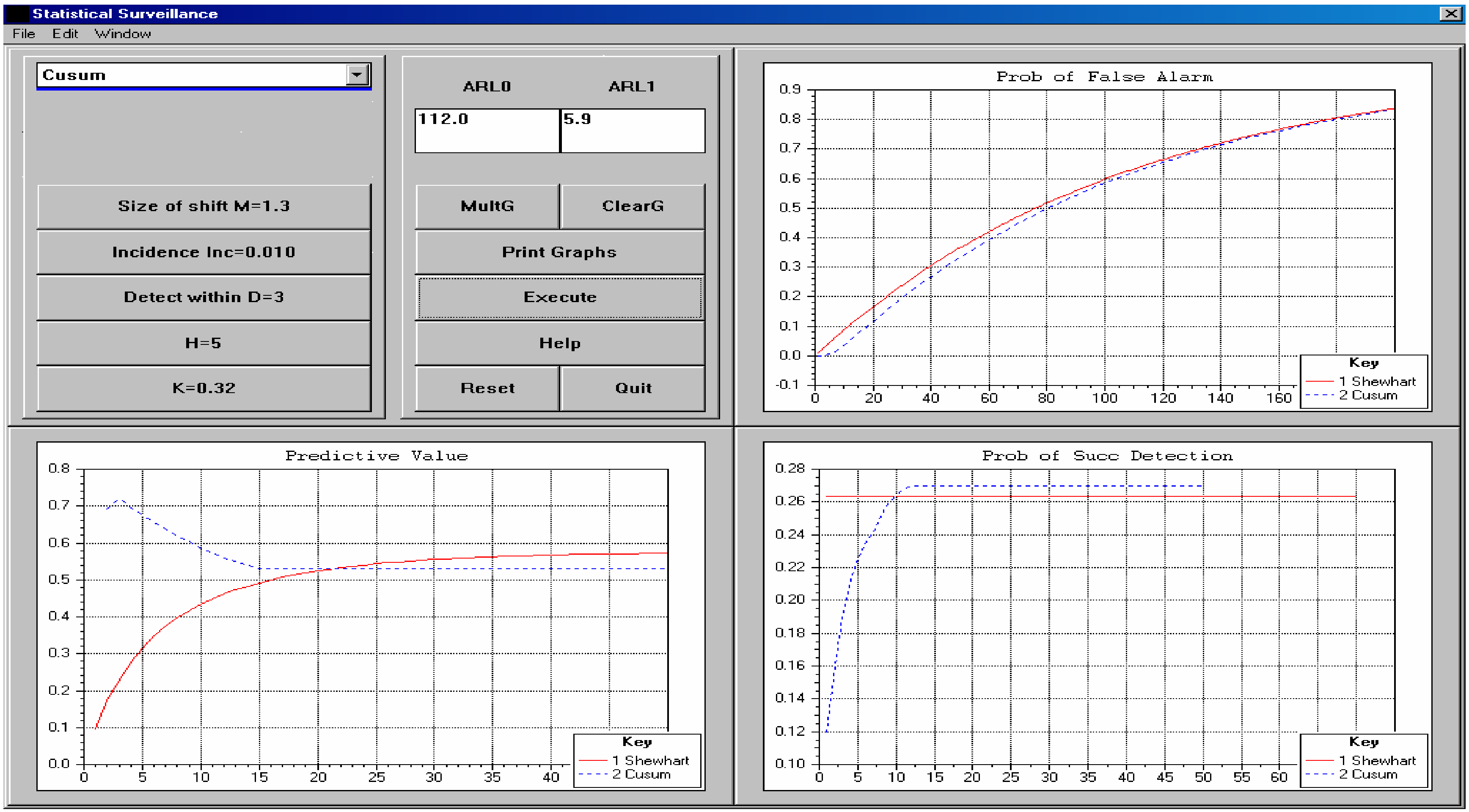
$$PSD(\tau, d) = P(t_A - \tau \le d \mid t_A \ge \tau).$$

Predictive value

$$\Pr(\tau \leq t \mid t_A = t)$$

The predictive value reflects the trust you should have in an alarm.

Computer program



Optimality

- ARL-optimality
- Expected Delay-optimality
- Minimax-optimality

- Frisén, M. and de Maré, J. (1991). Optimal surveillance. *Biometrika*, 78, 271-80.
- Frisén, M. (2003). Statistical Surveillance. Optimality and Methods., *International Statistical Review,* 71, 403-434.

ARL Optimality

- Minimal ARL^1 for fixed ARL^0
- Useless methods are ARL optimal
  - Frisén, M. (2003), Statistical Surveillance. Optimality and Methods. *International Statistical Review*, 71,403-434.
- Use only with care!

Minimal expected delay

- For a fixed false alarm probability

- The full likelihood ratio method LR is optimal
- EWMA Approximately optimal
  - for a certain value of the parameter
  - Frisén, M. and Sonesson, C. (2003): Optimal surveillance by exponentially moving average mehtods. Submitted.

Minimax Optimality

- Minimal expected delay
  for the worst value of τ
  and for the worst history of observations before τ
  - **CUSUM is minimax optimal**
  - Moustakides, G. V. (1986), "Optimal Stopping Times for Detecting Changes in Distributions," *The Annals of Statistics*, 14, 1379-1387.

- Shewhart Optimal for recent large change

Predictive value

A **constant predicted** value
makes the same kind of action appropriate
both for early and late alarms.

- Shewhart
  - Many early alarms.
  - These alarms are often false.
  - The predicted value of early alarms is very low.
  - Different actions motivated for early and late alarms

Syndromic surveillance

- Multivariate
- If the different symptoms change at the same time (or with a known time lag)
  - the multivariate situation is easily reduced to a univariate one
  - Wessman, P. (1998) Some Principles for surveillance adopted for multivariate processes with a common change point. Communications in Statistics. Theory and Methods, 27, 1143-1161.

Outbreak detection

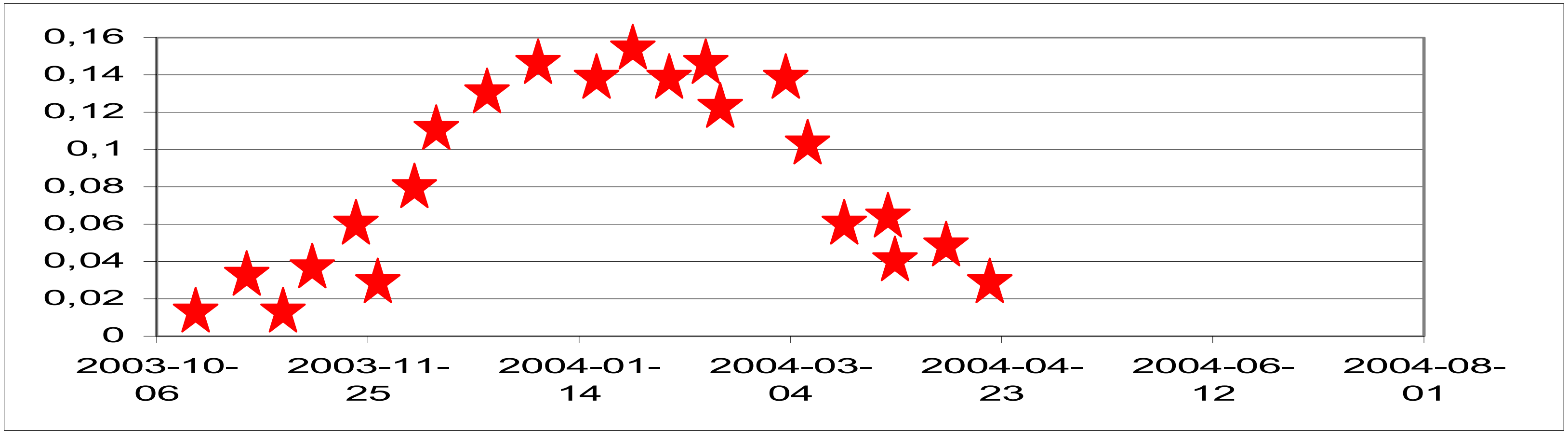
- Non-normal distribution
- Multivariate
  - Syndrome
  - Spatial
- Complicated changes

  - Frisén, M. (2003), Statistical Surveillance. Optimality and Methods., *International Statistical Review,* 71, 403-434.

Type of change

- Change from unknown baseline
- Gradual change
- Change in monotonicity

Outbreak



- Rise
- Decline

New robust method for detection of rise or decline

Maximum likelihood ratio

$$\left\{ x_s : \quad \frac{\max f(x_s | C)}{\max f(x_s | D)} \geq k' \right\}$$

- Frisén, M. (1986) Unimodal regression. *The Statistician,* **35,** 479-485.
- Andersson, E., Bock, D. and Frisén, M. (2002) Statistical surveillance of cyclical processes with application to turns in business cycles. *Submitted*.

Concluding remarks

- Robustness is important
- Knowledge of properties of a system is useful
- This involves many aspects
- Statistical measures which take care of time